

Designing Effective Professional Development: How do We Understand Teachers' Current Instructional Practices?

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Drawing on a review conducted of the resources that the mathematics education research community has developed while learning to support teacher learning, I direct attention to researchers' understanding of teachers' current practices. In particular, I argue that designers, facilitators, and researchers of professional development alike would benefit from understanding teachers' practices (a) as reasonable from teachers' perspectives, (b) in a way that can directly feed into the efforts of supporting teacher learning, and (c) as shaped by the institutional context of teachers' work.

Introduction

Designing effective professional development (PD) programs for mathematics teachers is a complex endeavour about which a lot remains to be learned (Borko, 2004). To explicate the complexity, I first discuss how it is deeply rooted in the demands related to teaching mathematics for understanding. I then argue that for PD interventions to be effective, the facilitators need to have both an understanding of teachers' current instructional practices and a way to build on those PD designs. Lastly, I use illustrations from several PD research studies to build an image of what might be involved in understanding teachers' practices in useful ways for the purposes of designing and facilitating effective PD programs.

For the past 15 years, an important goal for mathematics educators in the US has been to change the nature of mathematics teaching and learning in classrooms. Reformers have proposed substantial changes in the content and pedagogy of the K–12 mathematics curriculum, so that all students have the opportunity to learn more intellectually demanding mathematics. Among the important contributions of the reform efforts to this point is that they “shed light on the vital role played by teachers in educational change” (Llinares & Krainer, 2006, p. 439). The broad consensus about the critical role of teachers fuelled studies of classroom instructional practices that would support all students' development of the kinds of mathematical understanding that are the aim of the reform. A number of these studies suggest that the effective practices require that teachers build from their students' current reasoning while, at the same time, keeping in mind significant mathematical ideas that are the goal of instruction (e.g., Ball, 1993; Gravemeijer, 2004; Hiebert et al., 1997; Lampert, 2001). The forms of the envisioned instructional practices emphasise students' opportunities to engage in mathematically challenging tasks, maintaining the level of challenge as tasks are enacted in the classroom (e.g., Stein & Lane, 1996), and students' opportunities to communicate their mathematical thinking (e.g., Lampert, 2001).

The complexity of supporting mathematics teachers to develop such instructional practices has been documented by numerous investigations that focused on teacher PD (e.g., Cobb & McClain, 2001; Fennema, Carpenter, Franke, & Carey, 1993; Franke & Kazemi, 2001; Simon & Tzur, 1999). Researchers reported that even in cases when teachers were willing to collaborate and seemed engaged in the work-session setting, understanding children's reasoning was not always easy (Ball, 2001; Schifter, 2001). In

addition, teachers did not always see the use of their new knowledge in their classrooms as immediately obvious (Fennema et al., 1993; Zhao, Visnovska, Cobb, & McClain, 2006). Part of this complexity resides in the nature of the required teacher learning that targets changes in what Elmore (1996) called “the core of educational practice” – that is, the ways teachers think about the nature of knowledge, the nature of mathematics that would be beneficial for students to learn, as well as about their own and their students’ roles in teaching and learning (cf. Carpenter et al., 2004). Those conducting PD thus face a challenge in finding ways to support the teachers to revise the core assumptions of their practice and help them develop a need to change their classroom instruction. This is where understanding teachers’ current practices in a useful way comes to the foreground in the process of designing effective PD.

Intervening to Support Mathematics Teachers’ Learning

Designing PD programs that build on and benefit from teachers’ current instructional practices and, at the same time, are effective in pursuing a PD agenda is important for reasons parallel to those of building on students’ reasoning towards an instructional agenda in mathematics classrooms. On the one hand, there is little doubt that PD interventions should pursue their agendas, such as to focus on the key learning goals for teachers. On the other hand, my experiences when working with a group of middle school mathematics teachers convinced me that linking these goals to the participating teachers’ current practices so that the teachers could come to see them as beneficial was as important (e.g., Zhao et al., 2006).

The issues I discuss in this paper arose when I reviewed the research on teacher PD in mathematics, with a goal of gaining better insights into understanding teachers’ current practices and how they can be used effectively as a resource in designing and facilitating PD. Pragmatically, I concentrated on interventionist studies with the goal of supporting teachers to develop instructional practices centred in student’s mathematical reasoning. In particular, I tried to understand what guidance asking different questions and adopting different perspectives bring to an endeavour of supporting and understanding teacher learning. The studies I discuss in this paper are intended to serve as paradigmatic cases of pursuing specific types of research goals while drawing on a specific set of assumptions and perspectives. They enable me to raise issues of importance with respect to *understanding* teachers’ current practices, specifically understanding them (a) as reasonable from teachers’ perspective, (b) in a way that can directly feed into the efforts of supporting teacher learning, and (c) as shaped by institutional context of teachers’ work.

Understanding Teachers’ Practices as Reasonable from their Perspective

Although recommendations to view teachers’ instruction as reasonable are a repeating theme in teacher education literature (e.g., Leatham, 2006; McIntyre & Hagger, 1992; Thompson, 1992), developing such a view might often seem counterintuitive. This is true especially in cases when teachers’ instructional practices differ significantly from those advocated by the reform proponents. However, if we do not commit to see teachers’ current instruction as reasonable from their perspective we risk both (a) overlooking opportunities for supporting teachers in making their perspectives a worthwhile topic of guided reflection, and (b) positioning teachers as deficient, having little to bring to the new instructional practices they are expected to develop. The professional developers’ job then becomes filling the gaps between teachers’ current – “deficient” – instructional practices

and the envisioned ones. The problematic nature of this approach is well documented by the frustrations of both teachers who ended up participating in PD programs that were not justifiable within their current understanding of teaching and learning (e.g., Putnam & Borko, 2000), and professional developers who struggled to earn participating teachers' compliance and enthusiasm (e.g., Franke, Kazemi, Carpenter, Battey, & Deneroff, 2002). The resulting mismatch in professional developers' and participating teachers' views of ways to improve classroom mathematics instruction has been discussed in terms of incongruence in beliefs (e.g., Tillema, 1995) and changing teachers' beliefs has been repeatedly reported a challenging task (e.g., Thompson, 1992).

Simon and colleagues (Simon, 2000; Simon, Tzur, Heinz, Kinzel, & Smith, 2000) illustrated that if we want to take teachers' current instructional practices "as a valuable starting point, not as something to be replaced, but a useful platform on which to build" (McIntyre & Hagger, 1992, p. 271), understanding these practices as a coherent system, rather than a random conglomerate of teaching moves, is valuable. Their Mathematics Teacher Development (MTD) Project experiences suggest that approaches that succeed in taking teachers' current instructional practices as a PD starting point might significantly reduce problematic mismatches between researchers' expectations and teachers' actual participation in PD activities. The phenomenon of teachers' "constraining" beliefs might then be tackled by re-conceptualising the issue as a problem of PD design. To investigate whether this indeed is the case, two related issues arise for those interested in supporting teachers' development of new instructional practices: (a) How to see and explain the teachers' actions as reasonable from their perspective, and (b) How to design for PD that builds on teachers' current instructional practices towards a PD agenda rather than pursuing a gap-filling approach. In the work I reviewed, Simon and colleagues productively contributed to addressing the first question by generating *accounts of practice* (Simon & Tzur, 1999) – an adaptation of a case study methodology tailored to yield insights into an individual teacher's current perspective on teaching and learning while seeing this perspective as reasonable from the teacher's point of view.

Understanding Teachers' Practices *in order to* Support Teacher Learning

Simon and colleagues' focus in their study was on documenting perspectives that mathematics teachers held about teaching and learning and theorising these perspectives developmentally. This focus, as any particular focus, highlighted some aspects of teacher learning while it chose not to address other aspects. My goal in this section is to discuss the guidance that MTD Project research provided for both the design of further intervention and analysis of actual teachers' learning. As the researchers (Tzur, Simon, Heinz, & Kinzel, 2001) point out, we can think of guidance at three different levels.

On the broad level, categorising teachers with respect to their perspective on learning can help to highlight some of the key characteristics of instructional practices, development of which might be worth supporting. In this sense, the distinction between *perception-based* and *conception-based* perspectives that the researchers explicated provided a general direction for teacher development. Specifically, conception-based perspective stands for a common core of emergent and constructivist perspectives and its development requires a difficult shift from "we understand what we see" to "*we see what we understand*" (Simon et al., 2000, p. 585), a shift that can be counterintuitive to many teachers. On the other hand,

- (a) perception-based perspective is grounded in a view of mathematics as a connected, logical, and universally accessible part of an ontological reality. From this perspective, learning mathematics

with understanding requires learner's direct (firsthand) perception of relevant mathematical relationships. ... teaching involves creating opportunities for students to apprehend (perceive) the mathematical relationships that exist around them (Simon et al., 2000, pp. 579, 594).

This perspective is problematic in that the teachers often do not consider what students already have to know and be able to do *in order to* gain the valued insights. With respect to a perspective that underlies “traditional” teaching practices, developing a perception-based perspective suggests an important accomplishment. With respect to developing instructional practices that would support students' learning mathematics with understanding, further support of teachers' development of a conception-based perspective would be needed.

Fine-grained understanding of teachers' instruction as reasonable from teachers' perspectives is especially useful in both anticipating and analysing teachers' interpretations of designed activities. Explication of a perception-based perspective helped Simon and colleagues corroborate their observations. In the researchers' view, the teachers were not inquiring into the nature of their students' understanding in their daily instruction. Portraying teachers' decisions as reasonable from their perspective, however, helped the researchers to understand that from the teachers' perspective, they *were* basing their instruction on their students' reasoning. However, they were only doing it as long as students' reasoning corresponded – in teachers' view – to observable mathematical reality. Simon and colleagues stressed that the sense that the teachers were making of opportunities to explore students' reasoning both in their classrooms and in PD sessions was constrained by their current perspectives on teaching and learning. Promoting MTD Project teachers' inquiry into their students' reasoning would be likely interpreted by the teachers as something they already do in their classrooms and would therefore not lead to the envisioned changes in teachers' instructional practices.

In order to guide professional developers' decisions when planning specific interventions in response to teachers' actual participation a yet different grain-level of understanding teachers' actions is beneficial. I will refer to this as a *meso-level* of PD design. To guide the design effectively, this meso-level should, in my view, be specific enough to help developers discern aspects of teachers' current practices that might provide a springboard for further intervention. At the same time, it is an advantage if the grain size allows for consideration of how patterns in practices of the group, rather than individual teachers, are shaped. I now discuss each of these two points in more detail.

First, researchers working within constructivist, emergent, and situated paradigms concur that teachers' current instructional practices can and should serve as a basis on which to build in supporting teachers' further learning (Ball & Cohen, 1999; Kazemi & Franke, 2004; McIntyre & Hagger, 1992; Simon et al., 2000; Wilson & Berne, 1999). Pragmatically, they aim to design PD activities to promote participation that both engages teachers' current professional expertise and supports its transformation. MTD Project experiences illustrate that this is not a trivial task. For the teachers, further learning would involve a shift in paradigm with respect to development of mathematical knowledge. In what ways could teachers' current practices, oriented by a paradigm we want them to overcome, serve as a leverage in supporting the envisioned shift? I suggest that as designers of teacher PD with an ultimate goal of improving students' mathematical learning we need to understand teachers' current practices in ways that will allow us to answer this question. A systematic view of teachers' practices that would enable us to formulate revisable conjectures about ways of supporting teachers' learning on an ongoing basis would be of both theoretical and pragmatic value.

Second, although the usefulness of researchers' understanding was also a priority for Simon and colleagues, I would like to point to what I see as possible limitations of understanding teachers' practices solely in terms of individual teachers' underlying perspectives of mathematics teaching and learning. It has been documented that other aspects significantly influence teaching from teachers' point of view, often by shaping the setting in which teachers work. Aspects of teaching, like available instructional resources for use in classrooms (Cobb, McClain, Lamberg, & Dean, 2003; Remillard, 2005), teachers' views of student motivation and classroom misbehaviour (Dean, 2006; Visnovska, 2005; Zhao et al., 2006), and overall organizational aspects of the institutional contexts in which teachers work (Cobb et al., 2003; Elmore, 2000; Gamoran et al., 2003), all significantly shape how teachers approach teaching and learning. Each of these aspects constitutes a source of explanation to understand the rationality of teachers' instructional practices (Zhao, 2005) that remain in the background when the focus is on teachers' conceptions. More importantly, each of these may serve as a resource in designing starting points for PD that would capitalise on the teachers' current instructional practices. Several of these aspects of teachers' work point our attention to influences on teaching that are common across the participating teachers. From a perspective of a designer, this would allow for planning PD activities where current concerns of all teachers could become a topic of discussion. The teachers' individual responses to these common concerns could then provide the facilitator with the diversity of ideas on which to build in supporting teacher learning.

I would like to clarify that this broadening of the scope within which to understand teachers' practices is not motivated by a quest for an ultimate theoretical account. Others' PD experiences that I review suggest that we cannot expect that all teachers characterised as having developed a certain perspective on teaching and learning could be further supported in the same way. That is, in a way that would be independent of the institutional context of their work, instructional resources available in their schools, or major impediments to instruction as seen from teachers' perspective. As I illustrate in the following discussion of the Cognitively Guided Instruction (CGI) project experiences, this broadening of the scope has long been implicitly present, across the spectrum of adopted theoretical perspectives, in the designs of PD that could be claimed effective in supporting teacher learning.

Understanding Teachers' Practices as Profoundly Shaped by Institutional Context of their Work

I first introduce a CGI study (Fennema et al., 1996) conducted under a cognitive research paradigm. I chose the study based on a rich picture that the researchers provided of the concerns that played an important role in their design and research efforts. Concerns that related to the institutional context of teachers' school were treated as background issues and were not accounted for within the cognitive framework adopted for the study. Nevertheless, it would be hard to overlook the design efforts explicitly devoted to shaping the institutional context in which the teachers worked.

CGI: Research-based Knowledge for Teaching

CGI researchers developed their program in the mid 1980s to investigate how mathematics teachers may capitalise upon research-based knowledge in their classroom instruction. In terms of content, most of the CGI research work was grounded in a

substantial body of research that provided a consistent and coherent picture of the development of basic number concepts (Carpenter, 1985; Carpenter, Fennema, Franke, Levi, & Empson, 1999; Fuson, 1992). Over the years, CGI researchers engaged in a number of research and PD projects in which they collaborated with a variety of mathematics teacher groups. The teachers' active part in the PD was in deciding how to make use of the knowledge in the context of their own classroom instruction. The researchers conjectured that by providing teachers with an operationalised model of how children's thinking develops the teachers would become competent in identifying different forms of students' mathematical reasoning in their classrooms, as well as in planning appropriate follow up instruction that would capitalise on identified forms of reasoning.

The success of the PD efforts was framed in terms of changes in the individual teachers' beliefs and instruction. Findings from case studies led the researchers to conclude that "developing an understanding of children's mathematical thinking *can be* a productive basis for helping teachers to make the fundamental changes called for in current reform recommendations" (p. 403, emphasis added). Such studies served as an existence proof of what could be achieved with teachers through focusing on a research-based framework of student thinking, and provided insights into the specifics of achieved instructional changes. Teachers' knowledge of students' developmental processes and their ability to understand their students' reasoning were both framed as instrumental to the documented changes.

In terms of means that supported the discussed developments, the early CGI reports accordingly focused on two issues (a) a research-based model of student thinking, and (b) teachers' use of that model in their classrooms. It is important to clarify that *supporting collaborating teachers' learning* also included the following.

A CGI staff member and a mentor teacher were assigned to each school. Their responsibilities included participating in the workshops, visiting classrooms, engaging the teachers in discussions, and generally providing support as the teachers learned to base instruction on their students' thinking. Both staff members and the mentor teachers were trained to focus most of their interactions with teachers directly on children's thinking and its use. Insofar as possible, these interactions concerned specific children (Fennema et al., 1996, p. 409).

In its plan of action, the CGI program did not focus solely on cognitive aspects of teachers' learning. It involved significant interventions with both school principals and mathematics support staff based in the teachers' schools. In order to generate the proof of the usefulness of research-based knowledge to teachers' instruction, the researchers took seriously the *institutional context* within which teachers worked. In a very real sense, the CGI work involved designing for a particular institutional context that the researchers envisioned as supportive of teachers' learning. Yet, at this point, these considerations were conceptualised as a background for the project, rather than as key support for teachers' developing practices. The distinction is critical with respect to generalizability of the research findings, that is, with respect to the orientation the findings provide to designing and facilitating teacher PD programs. I clarify this issue when I discuss one of the more recent CGI studies, in which the researchers drew on situated theories of learning and used considerations related to institutional setting as resources for understanding teachers' current instructional practices. I draw on this study to corroborate further what I mean by usefulness of understanding teachers' practices on the *meso-level* of PD design.

CGI: The Case of Algebraic Reasoning

After years of experience with PD in context of early number concepts, Franke and colleagues (Franke, Carpenter, & Battey, in press; Franke et al., 2002) engaged in PD and

research efforts focusing on early algebraic thinking. Using their intimate understanding of CGI principles and findings, they aimed to support elementary teachers in enhancing students' ability to generate, use, represent, and justify generalizations about fundamental properties of arithmetic. As in their previous work, the researchers intended to do this by both supporting teachers in developing a model of students' development of algebraic reasoning, and by supporting teachers' development of practices that place their students' reasoning in the centre of classroom instruction. However, they came to view teachers' cognition as being inherently social, inseparable from the cultural and institutional aspects of teachers' work.

The case I discuss comes from a CGI collaboration with a group of teachers in one of the lowest achieving elementary schools in the state of California (Franke et al., 2002). The researchers intended to use discussions of student work as leverage in supporting teachers' appreciation of understanding students' algebraic reasoning in instruction. To the researchers' surprise and frustration, even after many work-sessions, student reasoning did not become something teachers wanted to learn about and use in their instruction: "All the teachers ... see is the answer and while this occurred initially in our earlier work the teachers quickly began to see on the paper and in their questioning what students did to solve the problem" (p. 28). The teachers continued to check for correctness of responses and did not find it useful to discuss in classrooms how different students arrived at their solutions. Instead, they requested that the researchers provide them with more "worksheets" for students to practice until they ceased making mistakes.

To support these teachers' learning effectively, the researchers needed to understand *why*, despite CGI efforts, it continued to be reasonable from the teachers' perspective to support their students' learning of early algebra by providing them with abundant opportunities to practice, and by correcting their mistakes. Simon and colleagues' focus on teachers' conceptions locates the source of the reasonableness of teachers' actions within individual teachers' cognition. According to analysis from such a viewpoint, the California teachers could be characterised as making instructional decisions within a traditional perspective, based on a view of algebra as a collection of rules and facts that can be best learned by repetition. Although such characterization might capture quite accurately teachers' actions at the time, it does not clarify why sustained efforts at supporting these teachers' change were not viable. This point is critical because, according to Franke and colleagues (2002), teachers initially focused on correctness and practice in the earlier CGI collaborations as well. However, supported by the CGI team, they soon came to appreciate student reasoning as an instructional resource. It appears that although providing a useful and specific orientation in terms of goals for teacher learning, Simon and colleagues' characterization of teachers' perspectives is not specific enough to guide the ongoing process of designing for teacher learning. The exclusively cognitive focus of this characterization seems insufficient to explain why the means of support that had proven effective earlier were not effective with the California teachers.

Franke and colleagues' (in press) analysis instead located the encountered PD difficulties in both content-specific demands on teachers' learning, and the institutional setting of teachers' work. This allowed the researchers to propose specific adaptations to the PD design that took into account the unique characteristics of the PD context. As an example, consider the content-specific dimension related to the institutional setting of teachers' work. It concerned the extent to which the content area in the focus of teachers' PD was central (or peripheral) within the curriculum used in the teachers' schools. The researchers documented that the emphasis that the curriculum put on a specific content

area had consequences for development of teacher's expertise in that area. Specifically, the differences manifested in both (a) the resources for PD work available in form of the teachers' current practices in the content area, and (b) the opportunities afforded for teachers' further learning in that area in their classrooms. To elaborate the first point, the number development directly related to the early grades curricula that were in place in the collaborating schools. However, the ideas of relational thinking and formulating conjectures that were central to the CGI model of development of students' early algebraic reasoning were not explicit aspects of the typical mathematics curricula. As such, these were not areas where teachers had many opportunities to hear their students work with the ideas, or to deepen their own algebraic understanding. Consequently, the teachers often lacked the confidence that they could master the content issues that might arise in their classrooms, and productively engage students in algebraic thinking.

To address the second point, the central position of early number development content provided teachers with plenty of opportunities to pose CGI word problems and ponder student solutions. In contrast, to make seemingly "extracurricular" algebraic reasoning an instructional focus in their classrooms, the teachers would have to develop ways to coordinate the mathematical content addressed explicitly in the required curriculum with supporting students in making generalizations, noticing relations, and justifying conjectures. Not surprisingly, this presented additional challenges for teachers' development of new instructional practices.

Researchers' understanding of critical content-related demands on teachers' developing instructional practices and how they relate to the institutional context of teachers' daily instruction oriented researchers' conjectures about viable means of supporting teachers' further learning. For example, the researchers reported that to help teachers develop knowledge about identifying opportunities for algebraic thinking, they brought examples of interactions they observed in teachers' classrooms to the group for discussion. In addition, they started to create structured opportunities for teachers to reflect on "where their own students are in their understanding of the various ideas of algebraic thinking" (Franke et al., in press), as students' progress in this content area did not feature on the district quarterly benchmark assessment. These adaptations, although open for further testing and modifications, serve as an example of the flexibility that understanding teachers' practices as situated in the cultural and institutional aspects of teachers' work affords those working with groups of mathematics teachers. Adopting this perspective seemed to enhance the CGI researchers' capacity to manoeuvre on the *meso-level* of PD design, where pragmatic decisions of how to proceed are informed by systematic ongoing analyses.

Summary

Although developmental approaches can help us delineate worthwhile end points for teacher learning, it appears that studies conducted under a situated paradigm are especially well positioned to develop valuable means for supporting teacher learning on the meso-level of design. On this level, understanding of teachers' practices yields resources that can directly feed back to PD designs. In this paper, I outlined an argument for usefulness of this level of understanding teachers' practices when designing and facilitating PD interventions. As an example, I discussed how CGI researchers adapted their PD design based on their ongoing analysis of the institutional context of teachers' work. However, detailed analysis would be required to understand how means of support based on these design resources contribute to teachers' development of new instructional practices. In

addition, understanding which aspects of teachers' practices would be most useful in feeding back to designs is an important question to answer.

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